Written Exam at the Department of Economics
Summer 2021
Economic Growth
Final Exam, June 3, 9am-noon

3-hour closed book exam. Answers only in English.

SOLUTION MANUAL

Falling ill during the exam
If you fall ill during an examination at Peter Bangsvej, you must:

- submit a blank exam paper.
- leave the examination.
- contact your GP and submit a medical report to the Faculty of Social Sciences no later than five (5) days from the date of the exam.

Be careful not to cheat at exams!
You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

1 Short essay questions

Question 1.1

Jones and Klenow (2016) provide an approach to comparing "welfare" across countries. They show that (consumption) inequality negatively influences country-level welfare . Why is that so?

Solution: The logic of the JK comparison is that expected utility is compared behind a "Rawlsian veil". Utility is as usual a concave function of consumption (and in addition it depends on leisure and life expectancy). Countries with a high level of (consumption) inequality are, behind the Rawlsian veil, "risky" from a welfare perspective, as there is a substantial risk of being born "poor", conditional on average living standards. Concavity of utility ensures the individual behind the veil is risk averse. As a result, inequality is "bad" for welfare according to this crieria.

Question 1.2

(a) The core principle which underlies the assumption of constant returns to capital and labor in the aggregate is the so-called replication argument. Explain what the replication argument is. (b) The conventional wisdom is that in order for GDP per capita to grow in the long run technology will have to improve. What is the core principle which leads to the insight that per capita growth requires technological change? Explain why this principle does not violate the replication argument.

Solution: (a) The replication argument is that if you replicate an existing firm you should be able to produce exactly the same as the pre-existing one. As a result, if you double capital and labor input, you should double output. But in so doing per capita income is left unchanged.

(b) Non-rivalry of ideas means that many entities (households, firms) can simultaneously employ an idea without limiting each other in utilizing the idea. Logically if you now imagine doubling capital, labor and ideas output more than doubles, which thus opens the possibility that per capita income can grow. This does not violate the replication argument precisely because of the principle of non-rivalry: When you replicated the firm you did not have to double technological knowledge because both firms simultaneously can utilize the same knowledge.

Question 1.3

Bloom et al (2020) show that the number of transistors on an integrated circuits has doubled roughly every other year since 1970. (a) Why do Bloom et al nevertheless think research productivity has fallen? (b) What model(s) of economic growth is consistent with their evidence? (c) Can Moore's law be a steady state? Explain.

Solution: (a) Since the 1970s the number of transistors have increased impressively. Indeed, Moore's law suggests an impressive annual growth in productivity of about 35%. What is often overlooked in popular discussions, however, is that this has been accomplished only at a cost: rising R&D input. Over the period Bloom et al find that R&D input has increased by a factor of 18. Considering a steady (but high) productivity growth rate it implies that R&D output per unit of input has declined, according to Bloom et al by about a factor of 8. (The exact numbers are not essential and it is not required that the students remembers them; its the qualitative argument and directions that matter.)

(b) For R&D productivity to be declining the ideas-production function needs to exhibit diminishing returns to scientific knowledge in generating new ideas (the notion that exiting ideas allow you to generate new ideas in the first place is sometimes referred to as the "standing on shoulders of giants" effect). This is the case if we believe in sufficiently strong so-called "fishing out effects", which like diminishing returns means that ideas are getting harder to discover the more you already know. If fishing out is present we violate constant returns to reproducible input in knowledge production and so the standard notion of "endogenous growth" fails. For growth to be sustained other inputs need to be growing, chief amoung the labor input which can work to counteract the tendency for diminishing returns: if ideas are getting harder to find a resolution could be to send out more people to search for new ideas. This takes us the another class of endogenous growth models known as "semi-endogenous" growth models.

(c) Yes. Provided there is continual population growth it is possible in theory to sustain growth despite fishing out. Of course, whether population growth is possible on a planetary scale in perpetuity is probably doubtful.

Question 1.4

Explain why it is challenging to estimate the causal effect of the adoption of industrial robots on labor market outcomes. Explain further the approach Acemoglu and Restrepo (2019) take to solve this problem.

Solution: Causality can go both ways when it comes to explaining a correlation between adoption of industrial robots and labor market outcomes. Robots might, for instance, make workers redundant, but a short supply of workers might also lead firms to adopt robots. Another example is an unobserved demand shock that affects both robot adoption and the labor market outcomes. To alleviate such endogeneity concerns, Acemoglu and Restrepo use a shift-share type instrument for robot adoption. The "shift" component is the industry-specific trends in robot adoption in Europe, and the "share" component is the ex ante industry mix in US commuting zones. Both can be thought of as exogenous to changes in labor market outcomes in individual commuting zones. Acemoglu and Restrepo do not have actual robot adoption data at the commuting zone level, so they estimate their baseline IV regression in reduced form.

Question 1.5

Explain why the arrival of entirely new products can bias conventional estimates of economic growth.

Solution: There are (at least) two distinct problems. First, a completely new product represent an expansion of consumption possibilities, and being new, we do not know how valuable this expansion is to consumers as we do not observe the associated consumer surplus. Second, new products only enter the statistics with a lag, not just because at least two periods of price data are needed to construct a price index, but in practice because it often takes many years from a new product becomes available to consumers until statistical agencies catch on and include it in the consumption baskets used to calculate inflation. The Hausman paper on the reading list gives the cell phone as an example: it took as much as 15 years from it was commercialized until it was included in US CPI. Such lags can potentially lead to a large upward bias in the price indices used to calculate GDP growth. The reason is that further innovation in new products often is swift after they first brought to the market, leading their prices to decline. If the GDP deflator misses these declines, they will tend to overestimate inflation, and consequently underestimate GDP growth.

Question 1.6

What is wage polarization and job polarization, and how does the task based model help us to understand the two phenomena?

Solution: Both terms refer to the fact that medium skill workers appear to have been losing out to both high skill and low skill workers in the past decades. In the task based model, skill biased technical change that increases the productivity level of high skill workers will extend the range of tasks performed by skill workers. In turn, this will push medium skill workers toward tasks previously undertaken by low skill workers in which they have less of a comparative advantage. As a result, their wage will decrease relative to both high- and low skilled workers, or, in other words, to wage polarization. Another mechanism that the task based model points to is automation of tasks previously produced my medium skill workers. That will result in employment polarization in the sense that employment in tasks associated with medium skill workers will decline relative to employment in other tasks. Again, wages of medium skill workers will be adversely affected, leading to wage polarization as well.

2 Intermediate goods and comparative development

Consider the following model formulated in discrete time:

$$Q_t = \bar{A}\left(K_t^{\alpha} L^{1-\alpha}\right)^{1-\sigma} X_t^{\sigma} \tag{1}$$

$$X_t = \bar{x} Q_t \tag{2}$$

$$Y_t = (1-\bar{x}) Q_t \tag{3}$$

$$K_{t+1} = sY_t + (1-\delta) K_t \tag{4}$$

where Q_t is gross output, X_t is intermediate good input, Y_t is GDP, $\bar{A}$ is a fixed productivity level, and K_t is capital. The parameters $\alpha, \sigma, \bar{x}$, and s are all assumed to be between 0 and 1.

Question 2.1

The assumptions of the model ensures the existence of a steady state. Show that GDP per worker in steady state can be written as:

$$y = \left\{\bar{A}(1-\bar{x})^{1-\sigma}\bar{x}^{\sigma}\right\}^{\frac{1}{(1-\alpha)(1-\sigma)}} \left(\frac{K}{Y}\right)^{\frac{\alpha}{(1-\alpha)}} \tag{5}$$

Solution:

$$\begin{aligned}
Y_t &= (1-\bar{x}) Q_t = (1-\bar{x}) \bar{A} \left(K_t^{\alpha} L^{1-\alpha}\right)^{1-\sigma} X_t^{\sigma} \\
Y_t &= (1-\bar{x}) \bar{A} \left(K_t^{\alpha} L^{1-\alpha}\right)^{1-\sigma} (\bar{x} Q_t)^{\sigma} \\
Y_t &= (1-\bar{x}) \bar{A} \left(K_t^{\alpha} L^{1-\alpha}\right)^{1-\sigma} \left(\frac{\bar{x}}{1-\bar{x}} Y_t\right)^{\sigma} \\
Y_t^{1-\sigma} &= \bar{A}(1-\bar{x})^{1-\sigma}\bar{x}^{\sigma} \left(K_t^{\alpha} L^{1-\alpha}\right)^{1-\sigma} \\
Y_t &= \left\{\bar{A}(1-\bar{x})^{1-\sigma}\bar{x}^{\sigma}\right\}^{\frac{1}{1-\sigma}} K_t^{\alpha} L^{1-\alpha} \\
Y_t^{1-\alpha} &= \left\{\bar{A}(1-\bar{x})^{1-\sigma}\bar{x}^{\sigma}\right\}^{\frac{1}{(1-\sigma)(1-\alpha)}} \left(\frac{K_t}{Y_t}\right)^{\alpha} L^{1-\alpha} \\
y_t &= \left\{\bar{A}(1-\bar{x})^{1-\sigma}\bar{x}^{\sigma}\right\}^{\frac{1}{(1-\sigma)(1-\alpha)}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}}
\end{aligned}$$

Question 2.2

Why and how does the steady state differ from the steady state in a standard Solow model? Explain the intuition (no math is required in your answer to this question).

Solution: There are no intermediate goods in the Solow model, meaning that $\sigma = 0$ and $\bar{x} = 0$. The present model is different from this benchmark for two reasons. First, the multiplier on $\bar{A}$is larger because there are now two adjustable inputs (capital and intermediate goods) that respond to productivity changes. Second, a deviation from the optimal allocation of gross output to intermediate goods and final goods, respectively, will reduce TFP.

Question 2.3

Suppose now that $\bar{x}$, s, δ, α, and σ are identical across countries, but $\bar{A}$ differs due to difference in technology or human capital. Derive an expression for the relative GDP per capita of two countries, denoted Country 1 and Country 2, as a function of their relative productivity levels, $A_1$and A_2, and the parameters of the model.

Solution: K/Y is determined by δ and s in steady state, so they are identical across countries. Consequently we have that $\frac{y_1}{y_2} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{(1-\sigma)(1-\alpha)}}$.

Question 2.4

a) What is the optimal value of $\bar{x}$ in the model outlined above? Explain how your result can be used to calibrate the value of σ, and what an empirically reasonable value of σ might be. b) What is a reasonable guess for the size of α, and why is it a reasonable guess?

Solution: The optimal $\bar{x}$ maximizes steady state income given by Equation (5). The simplest way to derive the optimum is to use a log transformation, which lead to the following first order condition:

$$\begin{aligned} \frac{\partial \ln y}{\partial \bar{x}} &= -\frac{1-\sigma}{(1-\sigma)(1-\alpha)}\frac{1}{1-\bar{x}} + \frac{\sigma}{(1-\sigma)(1-\alpha)}\frac{1}{\bar{x}} = 0 \\ \Leftrightarrow \sigma(1-\bar{x}) &= (1-\sigma)\bar{x} \\ \Leftrightarrow \bar{x} &= \sigma. \end{aligned}$$

Empirical evidence shows that $\bar{x}$ is between 0.4 and 0.6 in all countries with available data (Figure 1 in Jones 2011). If distortions to the share of gross output allocated to intermediate goods are randomly distributed with zero mean, it would imply that optimal $\bar{x}$ is 0.5, which in turn indicates that $\sigma = \frac{1}{2}$.

The standard assumption is that $\alpha = \frac{1}{3}$. The reason is that the income share of labor, $1-\alpha$, is around $\frac{2}{3}$ and quite stable over time in most countries.

Question 2.5

Use the proposed values for α and σ from your previous answer to quantitatively compare Country 1 and Country 2 assuming that $\frac{A_1}{A_2} = \frac{1}{2}$. Compare to a situation with $\sigma = 0$, and explain why intermediate goods and weak links are useful for explaining comparative development.

Solution: $\alpha = \frac{1}{3}$ and $\sigma = \frac{1}{2}$ means that he exponent $\frac{1}{(1-\sigma)(1-\alpha)}$ becomes equal to 3, such that:

$$\frac{y_1}{y_2} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

The income difference between the two countries is four times larger than their productivity difference, everything else being equal. The implication is that relatively small differences in, e.g., human capital endowments can result in relatively large income differences (human capital will be captured by A here). Such amplification mechanism makes it easier to reconcile the large income differences among countries with the much smaller observed variation in education levels. At a more general level, an amplification mechanism of this sort will allow countries to be relatively similar in their use of inputs and technology, and still be quite different in terms of income per capita.

3 Premature death in an OLG model

Consider an overlapping generations economy where economic activity extends into the infinite future. The economy is closed and all markets are competitive. All individuals survive at most for two periods. During the first period people work and decide on how much to save for retirement. During this period they also have an off-spring (costlessly), which will ensure the population always is populated. There is no population growth, so each young person "spawns" just one off-spring. The size of each generation is normalized to one for notational simplicity.

Getting to experience retirement is not guaranteed however. Only with probability π will the individual manage to experience retirement after which they die for sure, as in a standard Diamond model. With probability $(1-\pi)$ they die prematurely. This will leave unclaimed savings, since this fraction of people do not get to consume the savings they build up during their working years. We assume that this income flow is simply passed on to their off-spring as "accidental bequest".

Specifically, individuals maximize expected utility

$$\ln c_{1t} + \pi \ln c_{2t+1}, \tag{6}$$

The first period budget constraint is $c_{1t} + s_t = w_t + b_t \equiv I_t$ and $c_{2t+1} = (1 + r_{t+1})\, s_t$. The notation is c_i for consumption ($i = 1$ for consumption during youth and $i = 2$ for consumption during old age), s is savings of the young, and b_t is accidental bequest. For future reference note that

$$b_t = (1-\pi)(1 + r_t)\, s_{t-1}. \tag{7}$$

That is, expected (and since there is no aggregate uncertainty: average) bequest is the probability of premature death multiplied by the capitalized value of the savings of their “parents”.

Question 3.1

Show that savings of the young is given by:

$$s_t = \frac{\pi}{1+\pi}\left(w_t + b_t\right). \tag{8}$$

Solution:

$$\ln\left(I_t - s_t\right) + \pi \ln\left(\left(1 + r_{t+1}\right) s_t\right)$$

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$$\frac{1}{I_t - s_t} = \pi \frac{1}{s_t} \Longleftrightarrow s_t = \frac{\pi}{1+\pi} I_t = \frac{\pi}{1+\pi}\left(w_t + b_t\right)$$

Question 3.2

The amount of capital available for the economy in the next generation is determined by the savings of the currently young generation, just as in a standard Diamond model $K_{t+1} = s_t$. Show that the law of motion for capital can be written as:

$$K_{t+1} = s^w w_t + s^r \left(1 + r_t\right) K_t, \tag{9}$$

where $s^w = \frac{\pi}{1+\pi}$ and $s^r = \frac{\pi(1-\pi)}{1+\pi}$. (Hint: To figure out how to elimate bequest you need to use equation (7) and that current period savings of the young determines the next period capital stock).

Solution: Insert for savings

$$K_{t+1} = s_t = \frac{\pi}{1+\pi}\left(w_t + b_t\right)$$

since $b_t = (1 - \pi)\left(1 + r_t\right) s_{t-1}$ and $s_{t-1} = K_t$ we have

$$K_{t+1} = s_t = \frac{\pi}{1+\pi} w_t + \frac{\pi}{1+\pi}\left(1 - \pi\right)\left(1 + r_t\right) K_t$$

which is the stated result.

Question 3.3

We assume firms operate a standard Cobb-Douglas production technology:

$$Y_t = K_t^{\alpha} (A_t)^{1-\alpha}, \tag{10}$$

where $A_{t+1} = (1+g) A_t$ is exogenous technological change and the size of the labor force, recall, is normalized to one. Firm maximize profits:

$$Y_t - w_t - (1+r_t) K_t, \tag{11}$$

where we assume capital depreciates fully during a period. Solve the profit maximization problem of the firm and proceed to show that the law of motion of capital in efficiency units can be written as:

$$k_{t+1} = s_t = \frac{\theta}{1+g} k_t^{\alpha}, \tag{12}$$

where $\theta = \frac{\pi}{1+\pi}(1-\alpha) + \frac{\pi}{1+\pi}(1-\pi)\alpha$ and $k_t = K_t/A_t$.

Solution. The first order condition wrt to capital is

$$1 + r_t = \alpha K_t^{\alpha-1} A_t^{1-\alpha} = \alpha k_t^{\alpha-1}$$

and the wage needs to fulfill (by zero profits)

$$\frac{w_t}{A_t} = \frac{Y_t}{A_t} - (1+r_t) K_t = (1-\alpha) k_t^{\alpha},$$

where the last equality uses the first order condition wrt capital. The law of motion for capital can be stated

$$\frac{K_{t+1}}{A_t} \frac{A_{t+1}}{A_t} = \frac{\pi}{1+\pi} \frac{w_t}{A_t} + \frac{\pi}{1+\pi} (1-\pi)(1+r_t) \frac{K_t}{A_t}$$

using that $A_{t+1} = (1+g) A_t$ and what we have learned about wages and the real rate

$$k_{t+1} = \frac{1}{1+g} \left[\frac{\pi(1-\alpha)}{1+\pi} + \frac{\alpha\pi(1-\pi)}{1+\pi} \right] k_t^{\alpha},$$

which is the expression we aimed to find.

Question 3.4

Illustrate the transition diagram for the model. Does a steady state exist? Is it unique? Stable?

Solution: Looks exactly like standard Diamond model. There is a unique globally stable (non-trivial) steady state.

Question 3.5

What the real rate of return in the steady state? In recent decades the real rate of return has exhibited a secular decline. Discuss what the model suggests could be driving forces behind this decline? Compare with what the (empirically based) conventional wisdom is.

Solution. In the steady state we have (from the transition equation evaluated at the steady state)

$$\left(\frac{y}{k}\right)^{*} = \frac{1+g}{\theta} = \frac{(1+g)(1+\pi)}{\pi(1-\alpha)+\alpha\pi(1-\pi)}.$$

so

$$r^{*} = (1+g)\frac{\alpha(1+\pi)}{\pi(1-\alpha)+\alpha\pi(1-\pi)} - 1$$

The only clear-cut prediction from the present model is that slower growth should work to reduce the long-run real rate of interest. This is indeed a possible mechanism that the recent literature has pointed to in that a case can be made that productivity growth has declined over time, which may (or may not) be related to the Bloom et al (2020) finding, cf shorter questions, that R&D productivity is declining ultimately due to fishing out (which is related to the arguments made by Robert Gordon, which the students also has had on their readings as supplementary). But the predictions regarding the remaining parameters is more ambiguous. Take adult life expectancy. A higher life expectancy will work to increase the propensity to save, which serves to drive up the capital stock in the long-run and tends to drive down the real rate of interest. At the same time, however, longer adult lives means less accidental bequest. Since accidental bequest is a redistribution of income from non-saving individuals to savers (the young) longer lives works to reduce savings through this channel. Technically this all comes down to how changes in adult longevity influence $\theta = \frac{\pi}{1+\pi}(1-\alpha)+\alpha\frac{\pi}{1+\pi}(1-\pi) = \left(\frac{\pi}{1+\pi}\right)[(1-\alpha)+\alpha(1-\pi)]$.

$$\frac{\partial\theta}{\partial\pi} = \frac{(1-\alpha)+\alpha(1-\pi)}{(1+\pi)^{2}} - \left(\frac{\pi}{1+\pi}\right)\alpha$$

If we imagine α is very large the impact from less bequest can dominate. If so greater adult mortality works to increase the real rate (by lowering the capital stock). The reason is that the reduction in savings generated by a lower transfer counteracts (in this case) the higher propensity to save among the young that greater expected life span entails. In the literature, however, the conventional wisdom is that aging (i.e. greater adult survival) leads to more aggregate savings and thus a lower interest rate. In the model this arises if the capital share is not too large. As for the influence from the factor income distribution the effect is similarly ambiguous as it depends on the state of adult mortality. High mortality makes it more likely that a higher capital share on net increases savings (i.e. dominate the effect from lower wages). There is not much debate on the role played by the factor distribution of income in regards to long-run real rates. But another perspective would be that there is actually inequality in the present model (albeit does not matter for long run outcomes with respect to steady state prosperity). The reason is the uncertainty of life span which means some consumers receive bequest others don't. This generates income differences among the young and the differences depend in how important capital is in generating income. Higher α means more inequality. In the literature on long-run interest rates it is often argued that greater inequality may have generated lower interest rates (usually based on the argument that rich save more). Here inequality is thus more likely to be associated with lower interest rates if adult mortality is high (mediated by a positive impact from higher α). The present model is new to the students, which should be taken into account when evaluating the quality of the answer to this question.